Maintenance cost forecasting for a fleet of vehicles

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Abstract

Generally, when purchasing of a fleet of vehicles, after-sales service includes maintenance, repair and overhaul (MRO) contract : this contract guarantees the fleet availability during several years under usage constraints. MRO contract is an undeniable marketing argument for automobile manufacturers, but it is also an additional cost which must be assessed. Particularly as this contracts last far beyond the equipments useful life. To manage the spare parts warehouse and to to assess the costs of maintenance and repairs, manufacturer must be able to forecast the number of failures for a list of critical equipments, throughout the contract. Extensive researches on spare parts inventory management could be found. But in the case of MRO contract on a fleet of vehicles, this issue is particulary difficult. First of all, guaranteed maximum availability during several years requires information on both equipments reliability and vehicles usages. Furthermore, there were many factors of instability of the spare parts requirement over the years: on the one hand equipments may have various modes of ageing either an equipment may be subjected to several mode of ageing ; on the other hand, vehicle usages may vary significantly according to the mission profile.

The purpose of this article is forecast the spare parts requirement for a fleet of vehicles, over several years, using a simulation model based on statistical analysis of equipments inter-occurrences of failure and vehicle usages. By integrating this these two different cost drivers over the years, we provide a flexible tool of decision-making to prepare or manage a MRO contract for a fleet of vehicles.

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1 Introduction

Products degrade with age and usage and fail when they are unable to perform a required function, under given environmental and operational conditions *cf.* [1]. Therefore, customers need assurance that the product will perform satisfactorily over the useful life period (*i.e.* before ageing). A warranty is then a legal contract which requires that, after the sale, the seller agrees to remedy or compensate the buyer, for certain defects or failures in the product and for a specified time or amount of usage. See [2] for a review on warranty policies.

For highly engineered products, "custom-built" products or industrial products bought in lots as fleet of vehicles or aircrafts, it can be more economical for the buyer to outsource the maintenance to an external service agent (*e.g.* the manufacturer or an independent specialized company) through an outsourcing maintenance service contract. Furthermore, warranty contract guarantees free repair during the warranty period, but does not guarantee the availability. But in some cases, persistent product performance is a crucial stake of purchase. This is the case for nuclear plants, fleet of aircrafts or military vehicles. Maintenance, repair and overhaul (MRO) contrats provide customers with an outsourcing MRO service, to keep the product in service throughout the duration of the contract, but under usage constraints, see *cf.* [3] for an overview on maintenance outsourcing. Recently, due to the rapid technological development and fierce competition linked to the globalization, the manufacturers are under pressure to provide its clients with an maintenance service in their after-sale-servicing. As example, purchases of a fleet of tanks by a government may include ten years of maintenance with a guaranteed high level of availability.

For manufacturers, providing a maintenance service is an undeniable marketing argument, but it is also an additional cost. To set up a MRO contract it is necessary to gather not only information on systems reliability and maintenance but also on their usages*(i.e.* operational environment, operator skill and usage intensity,...). Furthermore, a system such as an automobile consists of many modular systems (*e.g.* powertrain, undercarriage, electrical,...), equipments (subsystems *e.g.* motor, wheels, batteries,...) and thousands of components that are supplied through an extensive suppliers network. Reliability performance of a systems, equipments or components is under control of suppliers based on their design and their quality production. Vehicle usage conditions are under control of drivers, according to operating environment, mission profil and usage intensity. In this context, the statement of a MRO contract for a fleet is especially challenging. Collection and analysis of equipments warranty claims and vehicle usage records can then help in the decision-making.

In this paper, we consider MRO contract proposed by a automobile manufacturer, to maintain a fleet of several hundred vehicles in service throughout the duration of the contract, but under specific usage conditions. The contract covers a list of equipments during more than 10 years. Vehicles usages are also periodically recorded in a vehicles-database. When equipments failed, usage between successive failures are recorded in an equipments-database. It should be noted that MRO contrats issues include both system reliability and maintenance but also usages data analysis *(i.e.* operational environment, operator skill and usage intensity,...). Extensive works has been done on reliability, maintenance and warranty management, see [4] and [5] for a detailed review. But very few address the issues related to mathematical aspects of MRO contract with the requirement of permanent availability for a fleet. Among these latter include [6],[7], [8] and references therein. Generally, the mean cumulative number of failures is estimated, but considering same usage or same rate of failure occurrences for all vehicles of the fleet. Here, vehicles were deployed over several years and consequently equipments have various degradation levels. Furthermore, vehicles were used intermittently in highly varied conditions (*e.g.* harsh and sloping terrain,...), for long periods (*e.g.* several dozen hours,...) and various mission profiles (*e.g.* normal or intensive usage).

Therefore, equipment lifetimes and vehicle usages have typically large variance. These various sources of variability are not controlled or not observed. But neglecting existing non-homogeneity can lead to large estimation errors. On the other side, more appropriate mathematical models lead to intractable computations.

Therefore, to forecast the number of failures during the MRO conrtatc, a simulation based model was developed incorporating both equipments failures database and vehicles usages database. Failures data and usages data have been processed specifically. Weibull distributions have been extensively used for modelling life data in medical or industrial applications. In particular, Weibull distributions exhibit decreasing, constant or increasing hazard function which makes them suitable for modelling complex failure data, cf. [9]. Three different models of Weibull distributions are used to modelling failure inter-occurrences : either a simple Weibull model or two-fold Weibull competing risk for homogeneous population of equipments with one or two failure modes, either a two-component mixture of Weibull for inhomogeneous population of equipment (each subpopulation has its own mode of failure). Vehicles usages regularly collected provide a signifiant, but non-homogeneous database. Indeed, according to the mission profile, vehicles can switch from the nominal usage mode of the fleet to a intensive one. Mission profile is not disclosed in the database, and yet it is an essential information for the management of the spare parts. Hence, a cluster analysis allowed us to obtain usage profiles for vehicles.

Simulation based approach is an alternative method to yield maintenance cost forecasting under realistic assumptions on equipment failures and vehicles usages. In addition, our approach provides a flexible decision-making tool before and after execution of the MRO contract for a fleet. The remainder of this paper is structured as follows. The problem is formalized in Section 2. The simulation based model is described in Section 3. Equipments inter-occurrence of failures and vehicles usages profiles modeling are discussed in following sections. This method is illustrated with alternative scenarios to forecast the number of equipments failed during the contact in Section 6. Section 7 summarizes the conclusions of this paper and discusses future extensions of this research.

2 Problem description

An equipment installed on a fleet of κ vehicles is considered. These vehicles were deployed over several years. They are used intermittently. A MRO contract is intended to maintain the fleet for t years and covers a list of critical equipments. After each failure, equipments are rapidly replaced by a new one. Free replacements are supplied under constraints on the average usages of the vehicles. Vehicles are equipped with various features differently used according to the purpose of the mission. Usage features are measured with different scales (*e.g.* motor usage in hours, battery usage in hours, kilometers travelled, number of usages of a feature,...). The manufacturer has then selected three scales of measure which will be stored in the vehicles database. Inter-occurrences of failures are measured with a scale specific to each equipment (*e.g.* hours for the motor, kilometers for wheels,etc).

2.1 Notation

For $1 \leq k \leq \kappa$, let us denote

$$
\begin{aligned}
X_{ki} \quad &= i^e \text{ failure inter-occurrence of an equipment installed on the } k\text{-th vehicle ;} \\
U_k(t) \quad &= \text{cumulated usage of the } k\text{-th vehicle, on } t \text{ years} \\
N_k(t) \quad &= \text{number of equipments failed for } k\text{-th vehicle, on } t \text{ years:} \\
&\mathbb{P}(N_k(t) \geq n) = \mathbb{P}(X_{k1} + \cdots + X_{kn} \leq U_k(t)) \\
N(t) \quad &= \text{total number of failures for } \kappa \text{ vehicles of the fleet, during } t \text{ years} \\
&= \sum_{k=1}^{\kappa} N_k(t).
\end{aligned}
$$

In this paper, the random variables (X_{ki}) are assumed to be independent and identically distributed. A failed equipment is replaced by a new one. Let F_X denote their common distribution.

2.2 Problem statement

The decision problem related to MRO contract include both to maximize the fleet availability for the costumer, and minimizing the maintenance cost for the manufacturer. Hence, this study is motivated by forecasting of the spare parts requirement. We'll have to compute

- the stock of spare parts n_{max}, for a required availability p:

$$\mathbb{P}\left(N(t) \leq n_{max}\right) = p; \tag{1}$$

- the expected number of failures:

$$\mathbb{E}\left[N(t)\right]. \tag{2}$$

To solve Eq. (1) and Eq. (2), the distribution of $N(t)$ has to be computed. A failed equipment is replaced by a new one, thus counting processes $(N_i(t))$ are renewal processes and $N(t)$ is a superimposed renewal processes *cf.* [10]. Generally, explicit analytical expression for $N(t)$ distribution is not available. Therefore to predict the number of equipments failed during the contract, we provided a simulation-based model from vehicles usages database and equipments failure database cf. Figure 1.

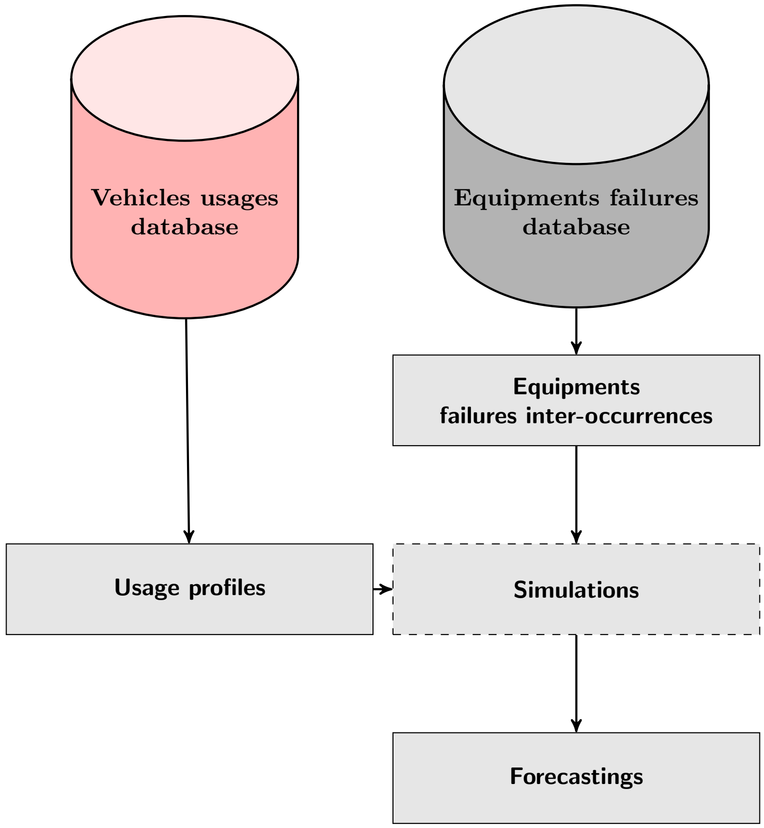


Figure 1: Forecasting of equipment failures of a fleet of vehicle using simulation based on vehicles usages and equipment failures

3 Simulation algorithm

The random nature of the number of equipment failures is related to the occurrence of equipment failures and the vehicle usages during the period *cf.* Algorithm 1.

Algorithm 1: Failures simulation algorithm

Initialisation: $N \leftarrow 0$
$U \leftarrow$ vehicle usage during a period
$X \leftarrow$ equipment usage until failure
while $X < U$ **do**
    $N \leftarrow N + 1$
    $U \leftarrow U - X$
    $X \leftarrow$ equipment usage until failure
end
return N

Model selection for both distribution of equipments inter-occurrence of failures X and vehicles usage U is made respectively from the equipment database and vehicle database. Effective data modelling requires a good understanding of properties of different models and tools and techniques to estimate parameters. In this respect, it is worth mentioning that usage between failures data and vehicle usages data are of a different nature and have to modelled in a specific manner.

4 Equipments failures inter-occurrence

Since more than five decades, Weibull distributions is extensively used for modelling life data in industrial applications. Indeed, Weibull distributions exhibit decreasing, constant or increasing hazard function which makes them suitable for modelling complex failure data cf. [9]. Here, vehicles were deployed to the field over several years and used in highly varied conditions (*e.g.* harsh and sloping terrain,...). Therefore, equipments lifetimes will have various failure rates. These various sources of variability are not controlled or not observed. From the equipments failure database analysis three models were considered.

- A single two-parameter Weibull model (SW) with probability density fonction :

$$f_X(x) = f_W(x|\beta, \eta) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} \mathrm{e}^{-\left(\frac{x}{\eta}\right)^{\beta}}, \tag{3}$$

  where $\beta > 0$ is the shape parameter and $\eta > 0$ the scale parameter. Here, equipment is assumed to have only one mode of failure.

- A two-fold Weibull competing risks model (CRW) with cumulative density fonction :

$$F_X(x) = 1 - (1 - F_W(x|\beta_1, \eta_1))(1 - F_W(x|\beta_2, \eta_2)) \tag{4}$$

  where F_W is the cumulative density function of a single Weibull distribution. Here, as for SW model, the population of equipments is homogeneous, but an equipment is subject to two different failure modes.

- A two-component mixture of Weibull distributions model (MW) with probability density fonction :

$$f_X(x) = \alpha\, f_W(x|\beta_1, \eta_1) + (1 - \alpha)\, f_W(x|\beta_2, \eta_2) \tag{5}$$

  Here, unlike SW model or CRW model, the population of equipments is not-homogeneous. Two main subpopulations with each their own failure mode can be distinguished (*e.g.* early or random failure, random or wear out failure, two designs or production processes,...).

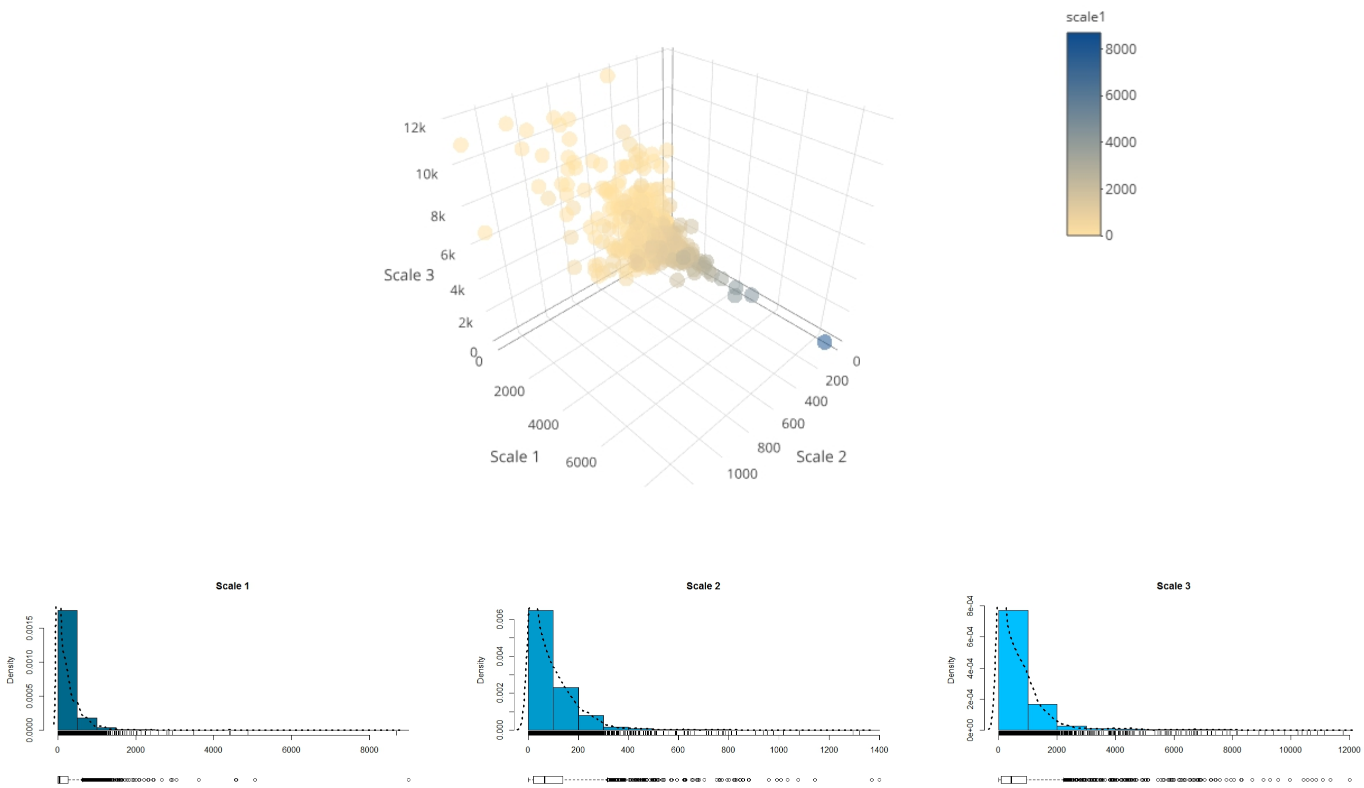
Weibull quantiles-quantiles plot (WQQP) can provide a quick and simply confirmatory method for data non-homogeneity *cf.* [11]:

- for single Weibull distribution, the WQQP has a straight line shape;
- for mixture of Weibull distributions, the WQQP has a single inflection point (S-shaped) with parallel asymptotes;
- for competing risks of Weibull distinctions, the WQQP has a convex shape.

Furthermore, WQQP provides crude estimates of model parameters, see [9] for computational details. These estimates are non precise nor robust but nonetheless can be used as starting values for alternative estimation algorithms. Different methods can be used for fitting a parametric model to data, but they have substantial difficulties in the setting of heavily censored data. Indeed equipment of critical equipment of vehicle are highly reliable, and consequently censoring rate are usually greater than 70%. With such levels maximum likelihood approaches (ML), Estimation-Maximization method include, provide estimate highly biased particulary for MW and CRW models, *cf.* [12] and [13]. Full Bayesian approach provides alternatives to ML methods. It consists of incorporating prior information on parameters to make inference [14]. Bayesian method require complex integrations for which Markov chain Monte Carlo algorithms (MCMC) are usually used ([**?**] is a good reference). For highly censored data MCMC methods appear to be deceptive in terms of convergence for MV et CM *cf.* [17] and [15]. Bayes method via Markov chain Monte Carlo algorithms is time consuming and convergence is difficult to be assessed, see [15]. In the context of highly censored data, the Bayesian restoration maximization (BRM) method provide an efficient alternative. It is a Bayesian bootstrap method which provides a sampling from the posterior distribution. Using an importance sampling technique, this sample is focused on the posterior mean. For computational details, see [17] for CRW model and [18] for MW model.

5 Vehicles usage profiles

Whether they are personal, commercial or military vehicles are equipped with various features differently used, according to the purpose of the mission. Usage of these features are measured by three different scales (*e.g.* hours, kilometers, number of uses,...). Therefore, usages of each vehicle are periodically stored in the database with three different scales. For confidentiality reasons they are denoted here scale 1, scale 2 and scale 3, see Figure 2.



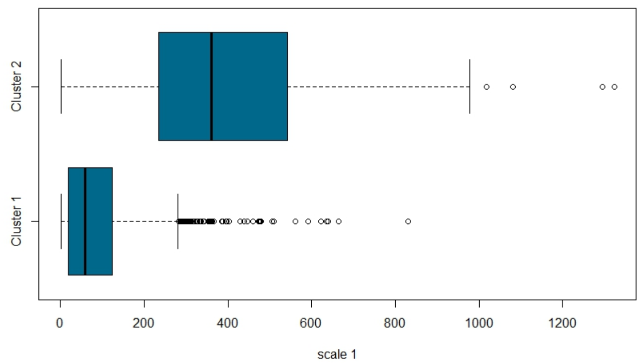
scale 1 scale 2 scale3

Figure 2: Scales of vehicles usages

Sometimes, some vehicles can switch from a the nominal usage mode of the fleet to a intensive one according to the mission profile. In case of conflict or intensive training for example. Such types of changes have an impact on the variability of the spare part request and must be integrate in the simulations. Unfortunately, mission profile is not disclosed in the vehicles database. To tackle this problem, a cluster analysis was used on the vehicles usages. The objective of clustering is to partition a population of items in homogeneous subgroups, such as items in the same subgroup are highly "similar", and items different subgroups are as different as possible *cf.* [19]. Two questions then arise : how to measure similarity ?

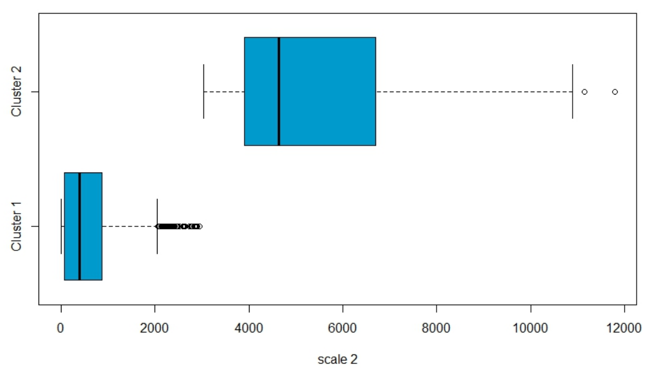
how many groups shall be selected ? From quantitative measures on items, the k-means algorithm with Euclidean metric as measure of similarity is the simplest clustering algorithm, see [20] for computational details. In our case, the aim is to identify vehicles profiles with usage intensities which are either normal or high. To ensure that profiles can be related to actual operating missions, the three usage units were used for clustering. As can be seen in Figures 3, 4, 5 and **??**, the two clusters match vehicles mission profiles with specific values of the three scales.

- Cluster 1: 96,5% of usages, low values with scale1 and scale 2, but some extreme values for scale 2. This profile is denoted $P1$.
- Cluster 2: 3,5% of usages, high values with scale1 and scale 2. This profile is denoted $P2$.



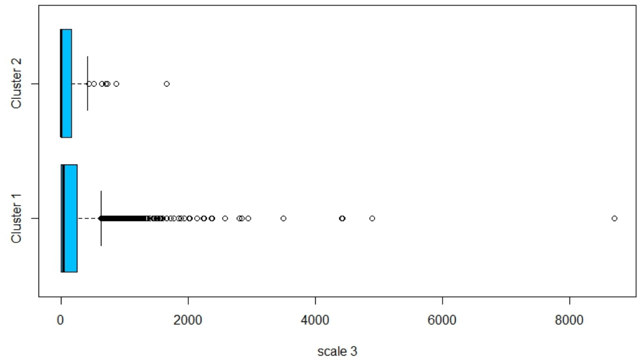
Clusters	%	mean	min	Quantile 25%	Median	Quantile 75%	max
Cluster 1	96.5%	83.9	1	19	59	124	831
Cluster 2	3.5%	425.8	2	235	360	540	1 323
All	100%	95.8	1	20	62	132	1 323

Figure 3: Vehicle usage unit 1 by clusters



Clusters	%	mean	min	Quantile 25%	Median	Quantile 75%	max
Cluster 1	96.5%	558.8	0	78	393	871	2 957
Cluster 2	3.5%	5 473.7	3 040	3 903	4 634	6 693	11 789
All	100%	729.8	0	84	420	937	11 789

Figure 4: Vehicle usage unit 2 by clusters



Cluster	%	mean	min	Quantile 25%	Median	Quantile 75%	max
Cluster 1	96.5%	189.3	0	0 3	9	252	8 692
Cluster 2	3.5%	115.1	0	0	3.5	169	1 655
All	100%	186.7	0	0	36	247	8 692

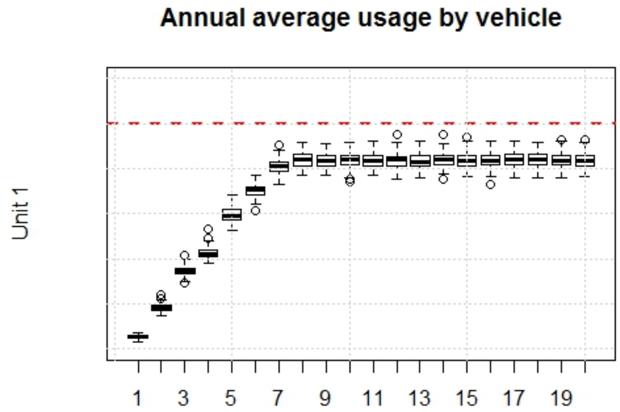
Figure 5: Vehicle usage unit 1 by clusters

6 Forecasting with various scenarios

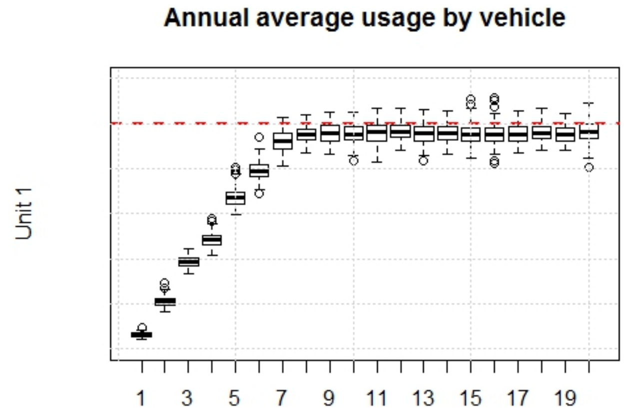
The simulation based model provides to forecasting of the failure number of equipment during the contract, which can be applied to determine spare parts ordering quantity. It is then a flexible decision-making tool which can be used for before and after execution of the MRO contract for a fleet. As an example, from several scenarios with different equipment failures and various vehicles usages profiles, a projection of the number of spare parts have been made for 20 years.

6.1 Average usage per vehicle

During a year, failed equipments are free replaced if the average usage per vehicle is below a threshold specified in the contract. Figure 6 shows that this threshold is not exceeded with a nominal usage, but it is however reached even with a small percentage of intensive usages. Therefore, a policy of use and management of the fleet by parks of vehicles should allow to optimize the cost : using specific usage profile by parks of vehicles



nominal usage profile $P1$



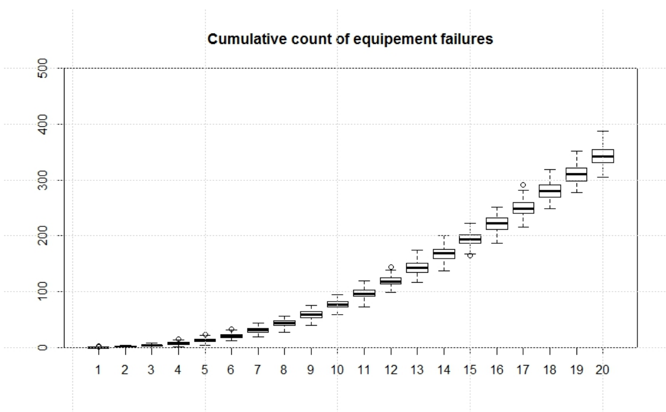
nominal and intensive profiles mixture $P1, P2$

Figure 6: Annual average usage by vehicle

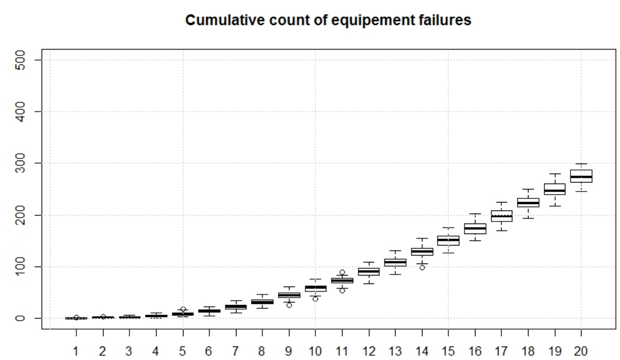
It should be noticed that the first eight years, correspond to the duration of the fleet deployment. All vehicles entering into service from the tenth year.

6.2 Homogeneous population of equipments with various usages profiles

The vehicle usage is a factor of instability of the spare parts requirement overs the years. From an homogeneous population of equipments with only one mode of failure, various scenarios of the vehicle usages have been investigated. Figure 7 shows the forecasting overs 20 years. The exponential behaviour of the curves is due to the deployment of the vehicles over the first nine years.



nominal and intensive profiles mixture $P1, P2$
$X \sim \mathcal{W}(\beta = 2\,;\, \eta = 2000)$

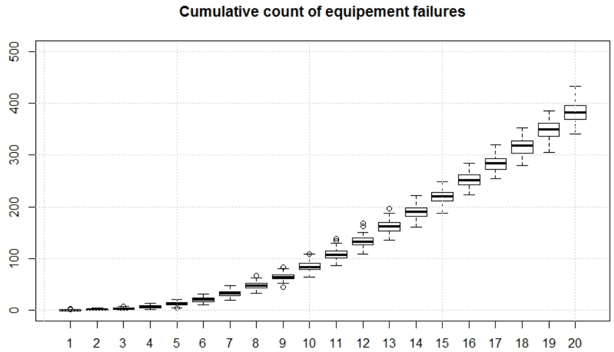


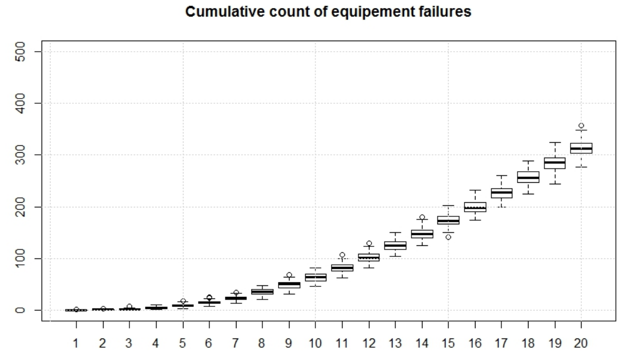
nominal usage profile $P1$
$X \sim \mathcal{W}(\beta = 2\,;\, \eta = 2000)$

Figure 7: Cumulative count of failures with simple Weibull

6.3 Non-homogeneous population of equipments with various usages profiles

The heterogeneity of equipments ageing is a factor of instability of the spare parts requirement overs the years. Various scenarios of mixing are investigated. Figures 8- 10 show forecasting overs 20 years with various vehicles usage profiles.

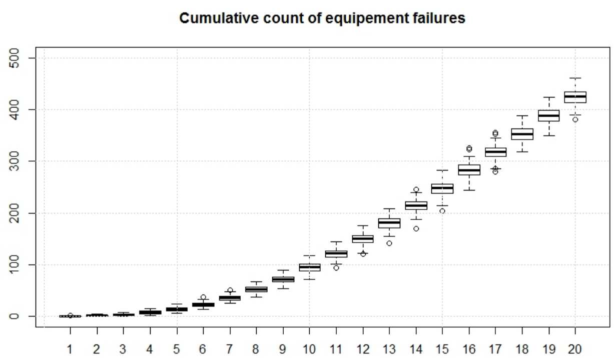


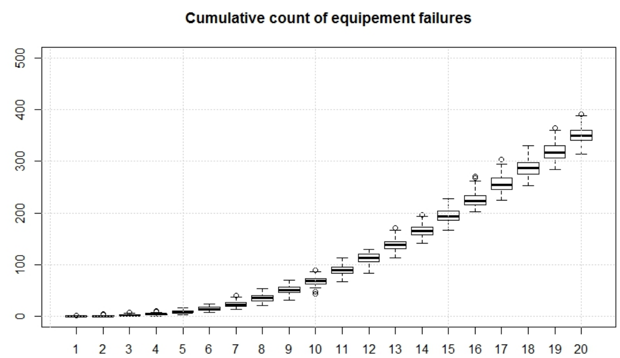


(a.1)
nominal and intensive profiles mixture $P1, P2$
$X \sim$ Mix Weibull
$\alpha_1 = 0.1$, $\mathcal{W}(\beta_1 = 4,\ \eta_1 = 1000)$
$\alpha_2 = 0.9$, $\mathcal{W}(\beta_2 = 2,\ \eta_2 = 2000)$

(a.2)
nominal usage profile $P1$
$X \sim$ Mix Weibull
$\alpha_1 = 0.1$, $\mathcal{W}(\beta_1 = 4,\ \eta_1 = 1000)$
$\alpha_2 = 0.9$, $\mathcal{W}(\beta_2 = 2,\ \eta_2 = 2000)$

Figure 8: Cumulative count of failures with various mixture of Weibull

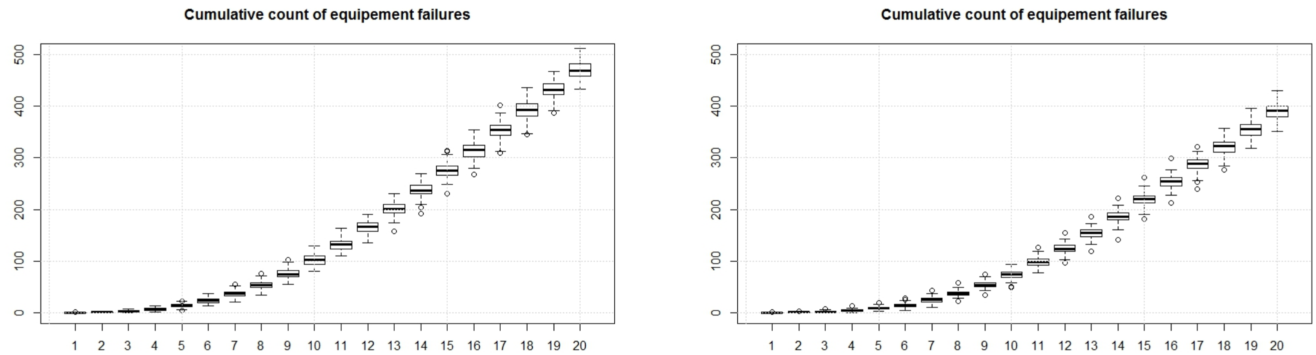




(b.1)
nominal and intensive profiles mixture $P1, P2$
$X \sim$ Mix Weibull
$\alpha_1 = 0.2$, $\mathcal{W}(\beta_1 = 4,\ \eta_1 = 1000)$
$\alpha_2 = 0.8$, $\mathcal{W}(\beta_2 = 2,\ \eta_2 = 2000)$

(b.2)
nominal usage profile $P1$
$X \sim$ Mix Weibull
$\alpha_1 = 0.2$, $\mathcal{W}(\beta_1 = 4,\ \eta_1 = 1000)$
$\alpha_2 = 0.8$, $\mathcal{W}(\beta_2 = 2,\ \eta_2 = 2000)$

Figure 9: Cumulative count of failures with various mixture of Weibull



(c.1)
nominal and intensive profiles mixture $P1, P2$
$X \sim$ Mix Weibull
$\alpha_1 = 0.3,\ \mathcal{W}(\beta_1 = 4,\ \eta_1 = 1000)$
$\alpha_2 = 0.7,\ \mathcal{W}(\beta_2 = 2,\ \eta_2 = 2000)$

(c.2)
nominal usage profile $P1$
$X \sim$ Mix Weibull
$\alpha_1 = 0.3,\ \mathcal{W}(\beta_1 = 4,\ \eta_1 = 1000)$
$\alpha_2 = 0.7,\ \mathcal{W}(\beta_2 = 2,\ \eta_2 = 2000)$

Figure 10: Cumulative count of failures with various mixture of Weibull

7 Conclusion

In this paper, we consider MRO contract proposed by an automobile manufacturer to maintain a fleet of several hundred vehicles. Throughout the duration of the contract, a maximum availability of the fleet is guaranteed. The contract covers repairs of a list of critical equipments during more than ten years. In this context the spare parts inventory management is particulary difficult due to several factors of instability over the time. Particularly, the heterogeneity of equipments ageing and heterogeneity of vehicles usages are two main cost drivers. To tackle this issue, a simulation model was used, based on statistical analysis of equipments inter-occurrences of failures and vehicle usages. Failure data and usages data were processed specifically. For modeling various ageing, three Weibull models was used. A simple Weibull for a homogeneous population of equipments with only one mode of failure, a two-fold Weibull competing risk when equipment have two modes of failure, and finally a two-component mixture of Weibull for heterogeneous population of equipments. Vehicles usages regularly collected provide a signifiant, but non-homogeneous database. Hence, a cluster analysis allowed us to obtain usages profiles for vehicles.

Simulation based approach yield spare part forecasting under realistic assumptions on equipment failures and vehicles usages. By integrating these two different cost drivers over the years, we provide a flexible tool of decision-making to prepare or manage a MRO contract for a fleet of vehicles.

Therefore, a simulation based model was developed incorporating both equipments failures database and vehicles usages database.

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